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Full Length Research Paper

Vibrations of elastically restrained rectangular plates

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In this study, an approximation method based on Fourier sine series were investigated for the vibration analysis of rectangular plates elastically restrained along all the edges. The transverse displacement of the elastic supported plate consisted of linear combination of Fourier sine series and an auxiliary polynomial function. In order to eliminate possible discontinuities; an auxiliary polynomial was used in Fourier solution function. For that, a displacement solution function that could be derived at least three times was adopted by letting series function to satisfy the governing differential equation for all the boundary conditions at every point. All the unknown Fourier expansion coefficients and natural frequencies of the plate were determined by employing the Galerkin discretization procedure. Unlike the existing techniques, the proposed method does not require a very tedious solution process, potential difficulties or non-linear hyperbolic functions. In the all performed calculations, the Kirchhoff plate theory, which is also called the classical plate theory, was employed. Several numerical examples were presented to demonstrate the accuracy and convergence of the current solutions.

Key words: Vibration of plate, elastically restrained plate, frequency parameters**.**

INTRODUCTION

Plates and plate-type structures have gained a special importance and notably increased engineering applications in recent years. A large number of structural components in engineering structures can be classified as plates. Typical examples in engineering structures are floor and foundation slabs, lock-gates, thin retaining walls, bridge decks and slab bridges. Plates are also indispensable in ship building and aerospace industries. The wings and a large part of the fuselage of an aircraft, for example, consist of a slightly curved plate skin with an array of stiffened ribs. The hull of a ship, its deck and its superstructure are further examples of stiffened plate structures. Plates are also frequently parts of machineries and other mechanical devices (Szilard, 2004).

It is worth mentioning that there exist altogether 21 different combinations of classical boundary conditions for a rectangular plate (Chakraverty, 2009). Although, elastically supported plates are very important in the application, there is limited study in the literature.

Transverse vibrations of rectangular plates with elastic boundary conditions have been studied in the literature, as reviewed by Leissa (1969, 1973); Gorman (1980, 2005) in the last three-forty decades.

Arbitrary non-uniform elastic edge restraints represent the most general class of boundary conditions for plate problems, and are encountered in many real-world applications. The vibrations of plates with this kind of boundary conditions, however, are rarely studied in the literature perhaps because there is a lack of suitable analytical or numerical techniques (Zhang and Li, 2009). Beam functions and combinations have been used in plate problem solutions to satisfy certain boundary conditions. A set of static beam functions have been developed to determine natural frequencies in transverse vibration of rectangular plates with elastic translational and/or rotational edges was calculated by employing the Rayleigh-Ritz method (Zhou, 1995, 1996). The free vibrations of thin orthotropic rectangular plates were analyzed using a set of static beam functions by the Rayleigh-Ritz method by Zhou (1999). Lee and Lee (1997) have adopted a new beam function set under point load as admissible functions and the Rayleigh-Ritz method to study the problem of the flexural vibration of thin, isotropic rectangular plates under elastic point

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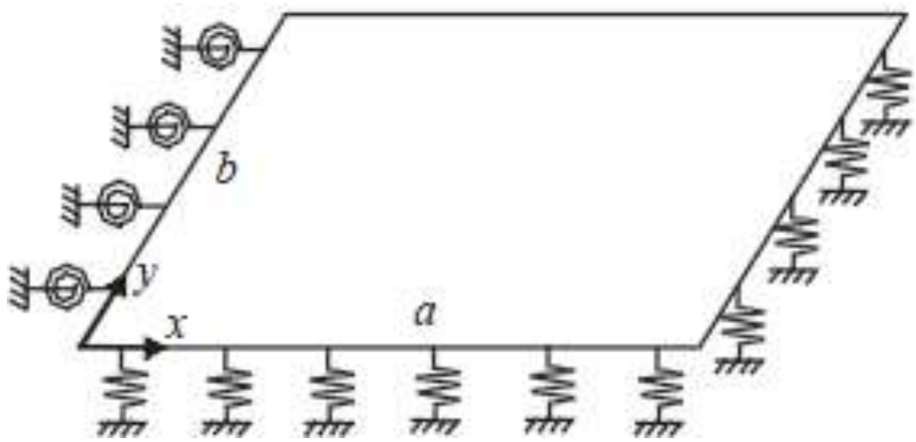

Figure 1. A rectangular plate elastically restrained.

support. A general solution of the vibration of an Euler–Bernoulli beam with arbitrary type of discontinuity at arbitrary number of locations has been presented using Heaviside function by Wang and Qiao (2007). Frequency equations in matrix form have been derived by Kim and Kim (2001) using Fourier sine series for calculating the natural frequencies of the beams with generally restrained boundary conditions by both translational and rotational springs.

Li and Daniels (2002) proposed Fourier series method with simply polynomial for transverse vibration of plates that are simply supported along a pair of opposite edges and elastically restrained with translational and rotational springs along the others in a general manner. Li (2004) also applied his own method for plates with general full elastically restrained boundary conditions and solved using the Rayleigh-Ritz method. Then, Du (2007) and Li et al. (2009) applied the same method also for the in-plane vibration problems of plates with fully restrained boundary conditions. W.L. Li is to be congratulated for their contribution in recent years to vibration analysis of elastically restrained plates. For the investigations on the vibrations of elastically restrained plates, W.L. Li's works can be examined.

Zarubinskaya and Horssen (2004) studied an initial-boundary value problem for a rectangular plate with general elastic supports alone its two opposite edges. They established a model which consisted of a rectangular plate with two opposite sides simply supported and the other sides attached to linear springs as a suspension bridge. Deformation, damage, crack etc. caused by vibrations can occur at the plates subjected to various loads (Yavuz et al., 2006.; Phoenix et al., 2006). Plate systems may be a more stable reduction of the deformations (Yavuz et al., 2006; Morgül and Küçükrendeci, 2008).

In this study, vibrations of plates with boundary conditions of elastic along full edges were studied. Deflections function was expressed as the combination of a Fourier sine series and an auxiliary polynomial. Solution function as employed by Li (2002) has been adopted for plates with fully elastic edges. Frequency parameters of plate were calculated for different plate parameters. Solutions obtained for elastically restrained plate were compared with related references.

SOLUTION METHOD FOR VIBRATION OF A RECTANGULAR PLATE

Consider a rectangular plate linear and rotational springs restrained along any edge, $x = 0$, $x = a$, $y = 0$ and $y = b$ illustrated in Figure. For clarity, only some the rotational and linear springs were shown. The governing differential equation for free vibration of a plate was given by:

$$D\nabla^4 w(x,y) - \rho h \omega^2 w(x,y) = 0 \tag{1}$$

Where $w(x,y)$ is the flexural displacement, ω is the angular frequency, D is the flexural rigidity, ρ is the mass density, h is the thickness of the plate and ∇^4 is the square of the laplacian operator. D and ∇^4 were expressed as:

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{3}$$

In terms of flexural displacements, the bending and twisting moments, and transverse shearing forces can be expressed as:

$$M_x = -D\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \nu\frac{\partial^2 w(x,y)}{\partial y^2}\right) \tag{4}$$

$$M_y = -D\left(\frac{\partial^2 w(x,y)}{\partial y^2} + \nu\frac{\partial^2 w(x,y)}{\partial x^2}\right) \tag{5}$$

$$Q_x = -D\left(\frac{\partial^3 w(x,y)}{\partial x^3} + (2-\nu)\frac{\partial^3 w(x,y)}{\partial x \partial y^2}\right) \tag{6}$$

$$Q_y = -D\left(\frac{\partial^3 w(x,y)}{\partial y^3} + (2-\nu)\frac{\partial^3 w(x,y)}{\partial x^2 \partial y}\right) \tag{7}$$

The boundary conditions along the elastically restrained edges can be expressed as:

$$k_1 w = Q_x, \quad \frac{K_0 \partial w}{\partial x} = -M_x, \quad at\ x = 0, \tag{8}$$

$$k_2 w = -Q_x, \quad \frac{K_1 \partial w}{\partial x} = M_x, \quad at\ x = a, \tag{9}$$

$$k_3 w = Q_y, \quad \frac{K_3 \partial w}{\partial y} = -M_y, \quad at\ y = 0, \tag{10}$$

$$k_4 w = -Q_y, \quad \frac{K_4 \partial w}{\partial y} = M_y, \quad at\ y = b, \tag{11}$$

Where k_1, k_2 are the linear stiffness's, K_1, K_2 are the rotational stiffness's of the elastic supports along $x = 0$ and a respectively, k_3, k_4 are the linear stiffness's, K_3, K_4 are the rotational stiffness's of the elastic supports along $y = 0$ and b respectively. Equations 8 to 11, describes a general boundary condition from which all the familiar homogeneous boundary conditions can be directly obtained by accordingly choosing the spring stiffness to be an extremely large or small number.

The current plate problem can be defined as boundary-value problem together with the differential equation, specific boundary conditions and physical properties. Fourier series approach consist of trigonometric sine expressions and polynomial was simply used for the boundary-value problem to be solved. Detailed information was presented about the Fourier series with polynomial by Li et al. (2002, 2004, 2009). The Fourier series simply represents a residual or conditioned displacement function that is periodic continuous and has at least three continuous derivatives everywhere. Based on the same consideration, the solution for the current plate problem can be written as Equation (12):

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{(A_{mn} \sin \lambda_{am} x + p(x)) \\ (B_{mn} \sin \lambda_{bn} y + p(y))\} \tag{12}$$

Where $p(x)$ and $p(y)$ denotes a polynomial which was determined below. The polynomials was particularly introduced to satisfy:

$$w(0,y) = \alpha_1 (B_{mn} \sin \lambda_{bn} y + p(y)), \tag{13}$$

$$w(a,y) = \alpha_2 (B_{mn} \sin \lambda_{bn} y + p(y)), \tag{14}$$

$$w''_{xx}(0,y) = \beta_1 (B_{mn} \sin \lambda_{bn} y + p(y)), \tag{15}$$

$$w''_{xx}(a,y) = \beta_2 (B_{mn} \sin \lambda_{bn} y + p(y)), \tag{16}$$

and

$$w(x,0) = \alpha_3 (A_{mn} \sin \lambda_{am} x + p(x)), \tag{17}$$

$$w(x,b) = \alpha_4 (A_{mn} \sin \lambda_{am} x + p(x)), \tag{18}$$

$$w''_{yy}(x,0) = \beta_3 (A_{mn} \sin \lambda_{am} x + p(x)), \tag{19}$$

$$w''_{yy}(x,b) = \beta_4 (A_{mn} \sin \lambda_{am} x + p(x)), \tag{20}$$

The lowest order polynomial that satisfies Equations 13 to 20 was shown thus:

$$p(x) = \frac{\alpha_1 (a-x)}{a} + \frac{\alpha_2 x}{a} \\ - \frac{\beta_1 (2a^2 x - 3ax^2 + x^3)}{6a} + \frac{\beta_2 (x^3 - a^2 x)}{6a} \tag{21}$$

$$p(y) = \frac{\alpha_3 (b-y)}{b} + \frac{\alpha_4 y}{b} \\ - \frac{\beta_3 (2b^2 y - 3by^2 + y^3)}{6b} + \frac{\beta_4 (y^3 - b^2 y)}{6b} \tag{22}$$

or in vector form:

$$p(x) = \zeta(x)\bar{\alpha}_x, \tag{23}$$

$$p(y) = \zeta(y)\bar{\alpha}_y, \tag{24}$$

Where:

$$\bar{\alpha}_x = \{\alpha_1 \quad \alpha_2 \quad \beta_1 \quad \beta_2\}^T \tag{25}$$

$$\bar{\alpha}_y = \{\alpha_3 \quad \alpha_4 \quad \beta_3 \quad \beta_4\}^T \tag{26}$$

$$\zeta(x) = \left\{\begin{matrix} \frac{(a-x)}{a} \\ \frac{x}{a} \\ -\frac{(2a^2 x - 3ax^2 + x^3)}{6a} \\ \frac{(x^3 - a^2 x)}{6a} \end{matrix}\right\}^T \tag{27}$$

$$\zeta(y) = \left\{\begin{matrix} \frac{(b-y)}{b} \\ \frac{y}{b} \\ -\frac{(2b^2 y - 3by^2 + y^3)}{6b} \\ \frac{(y^3 - b^2 y)}{6b} \end{matrix}\right\}^T \tag{28}$$

From the equations highlighted earlier, the boundary conditions, vectors $\bar{\alpha}_x$ and $\bar{\alpha}_y$ was determined as follows:

$$\bar{\alpha}_x = \sum_{m=0}^{\infty} \boldsymbol{H}_{nx}^{-1} \boldsymbol{Q}_{mx}^{n} \boldsymbol{A}_{mn} \tag{29}$$

Table 1. Frequency parameters for a C-S-C-S square plate.

Solution	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
Current	28.91	54.6476	69.3733	93. 91	102.0457	128.5865
Li (2002)	28.9514	54.7498	69.3275	94.6142	102.239	129.099
Leissa (1969)	28.9	54.8	69.2	94.6	102.2	129.1
Chakraverty (2009)	28.950	54.873	69.327	94.703	103.71	
Low et al.(1998)	28.97	54.77	69.40	94.71		

Table 2. Frequency parameters for a S-S-F-S square plate.

Solution	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
Current	11.5885	27.6851	41.1179	59.2032	62.9308	90.4242
Li (2002)	11.6859	27.7971	41.2308	59.2435	62.3701	90.5177
Leissa (1969)	11.68	27.76	41.20	59.07	61.86	90.29
Leissa (1973)	11.6845	27.7563	41.1967	59.0655	61.8606	90.2941
Chakraverty (2009)	11.684	27.757	41.220	59.360	62.461	-

$$\bar{\boldsymbol{\alpha}}_{\boldsymbol{y}} = \sum_{\boldsymbol{n}=\boldsymbol{0}}^{\infty} \boldsymbol{H}_{\boldsymbol{ny}}^{-\boldsymbol{1}} \boldsymbol{Q}_{\boldsymbol{my}}^{\boldsymbol{n}} \boldsymbol{B}_{\boldsymbol{mn}} \tag{30}$$

Where $\boldsymbol{H}_{\boldsymbol{nx}}^{-\boldsymbol{1}}, \boldsymbol{H}_{\boldsymbol{ny}}^{-\boldsymbol{1}}$, $\boldsymbol{Q}_{\boldsymbol{mx}}^{\boldsymbol{n}}$ and $\boldsymbol{Q}_{\boldsymbol{my}}^{\boldsymbol{n}}$ was determined respectively at Equations 31 to 34 as highlighted. Substituting equations 23, 24, 29 and 30 into equation (12), one immediately obtains the deflection function as Equation (35):

$$H_{nx} = \begin{bmatrix} \frac{k_1}{D} + \frac{(2-\nu)\lambda_{bn}^2}{a} & -\frac{(2-\nu)\lambda_{bn}^2}{a} & \frac{(2-\nu)\lambda_{bn}^2 a}{3} - \frac{1}{a} & \frac{(2-\nu)\lambda_{bn}^2 a}{6} + \frac{1}{a} \\ -\frac{(2-\nu)\lambda_{bn}^2}{a} & \frac{k_2}{D} + \frac{(2-\nu)\lambda_{bn}^2}{a} & \frac{(2-\nu)\lambda_{bn}^2 a}{6} + \frac{1}{a} & \frac{(2-\nu)\lambda_{bn}^2 a}{3} - \frac{1}{a} \\ \frac{K_1}{Da} - \nu\lambda_{bn}^2 & -\frac{K_1}{Da} & \frac{K_1 a}{3D} + 1 & \frac{K_1 a}{6D} \\ -\frac{K_2}{Da} & \frac{K_2}{Da} - \nu\lambda_{bn}^2 & \frac{K_2 a}{6D} & \frac{K_2 a}{3D} + 1 \end{bmatrix} \tag{31}$$

$$H_{ny} = \begin{bmatrix} \frac{k_3}{D} + \frac{(2-\nu)\lambda_{am}^2}{b} & -\frac{(2-\nu)\lambda_{am}^2}{b} & \frac{(2-\nu)\lambda_{am}^2 b}{3} - \frac{1}{b} & \frac{(2-\nu)\lambda_{am}^2 b}{6} + \frac{1}{b} \\ -\frac{(2-\nu)\lambda_{am}^2}{b} & \frac{k_4}{D} + \frac{(2-\nu)\lambda_{am}^2}{b} & \frac{(2-\nu)\lambda_{am}^2 b}{6} + \frac{1}{b} & \frac{(2-\nu)\lambda_{am}^2 b}{3} - \frac{1}{b} \\ \frac{K_3}{Db} - \nu\lambda_{am}^2 & -\frac{K_3}{Db} & \frac{K_3 b}{3D} + 1 & \frac{K_3 b}{6D} \\ -\frac{K_4}{Db} & \frac{K_4}{Db} - \nu\lambda_{am}^2 & \frac{K_4 b}{6D} & \frac{K_4 b}{3D} + 1 \end{bmatrix} \tag{32}$$

$$Q_{mx}^n = \left\{ \lambda_{am}^3 + (2-\nu)\lambda_{bn}^2 \lambda_{am} \quad (-1)^{m+1}[\lambda_{am}^3 + (2-\nu)\lambda_{bn}^2 \lambda_{am}] \quad \frac{K_1}{D}\lambda_{am} \quad (-1)^{m+1}\frac{K_2}{D}\lambda_{am} \right\}^T \tag{33}$$

$$Q_{mx}^n = \left\{ \lambda_{am}^3 + (2-\nu)\lambda_{bn}^2 \lambda_{am} \quad (-1)^{m+1}[\lambda_{am}^3 + (2-\nu)\lambda_{bn}^2 \lambda_{am}] \quad \frac{K_1}{D}\lambda_{am} \quad (-1)^{m+1}\frac{K_2}{D}\lambda_{am} \right\}^T \tag{34}$$

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\left(\sin\lambda_{am}x + \zeta(x)\boldsymbol{H}_{\boldsymbol{nx}}^{-\boldsymbol{1}}\boldsymbol{Q}_{\boldsymbol{mx}}^{\boldsymbol{n}}\right)\left(\sin\lambda_{bn}y + \zeta(y)\boldsymbol{H}_{\boldsymbol{ny}}^{-\boldsymbol{1}}\boldsymbol{Q}_{\boldsymbol{my}}^{\boldsymbol{n}}\right) \tag{35}$$

Where is $C_{mn} = A_{mn}B_{mn}$. Substituting equation 35 into equation 1 and using the Galerkin discretization procedure, one finally obtains:

$$(\boldsymbol{K} - \omega^2\boldsymbol{M})\boldsymbol{C} = 0 \tag{36}$$

Equation 36 is an eigenvalue-eigenvector problem. Consequently, the frequencies $\boldsymbol{\omega}$ of elastically restrained plate and the unknown expansion coefficients $\boldsymbol{C}$ of the Fourier series were obtained from equation 36. Then, frequency parameters for plate with elastic edges were calculated from Equation 37:

NUMERICAL RESULTS

First, let us consider a square plate that is clamped along $x = 0$ and simply supported $y = 0$ and b. The clamped boundary condition was easily generated by simply setting the stiffness's of the all springs equal to a very large number ($k_{1,2} = K_{1,2} = 10^{10}$). The simply supported boundary condition was easily generated by simply setting the stiffness's of the linear springs equal to a very large number ($k_{1,2}a^3/D = 10^{10}$), and the stiffness's of rotational springs equal to a very small number ($K_{1,2} = 0$). The first nine frequency parameters for C-S-C-S (Clamped-Simple-Clamped-Simple) were illustrated in Table 1. It was seen that the developed method led to a correct solution under M = N = 4 terms conditions.

The next example also deals with a familiar boundary condition: simply supported along $x = 0$, $y = 0$, $y = b$ and free along $x = a$. This boundary condition (S-S-F-S, Simple-Simple-Free-Simple) is readily represented by setting $K_{1,2,3,4} = k_2 = 0$, $k_1a^3/D = 10^{10}$ and $k_3a^3/D = k_4a^3/D = 10^{10}$. The six lowest frequency parameters obtained in the Fourier expansion were given in Table 2 under M = N = 4 terms conditions. It is seen that the current solution shows agreement with the references. The frequency parameters for C-S-E-S

Table 3. Frequency parameters for a C-S-E-S square plate.

Solution	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
Current	19.4114	40.7869	44.8337	67.3533	82.2153	92.4832
Li (2002)	19,4025	40,7803	44,8252	67,1697	81,6234	92,5912

Table 4. Frequency parameters for a S-S-S-S square plate.

Solution	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8	Ω_9
Current	19.7273	49.3242	49.3242	78.8848	98.5628	98.5628	128.1362	128.1362	167.6745
Li (2004)	19.74	49.35	49.35	78.96	98.70	98.70			
Low et al.(1998)	19.74	49.35	49.35	78.96					
Chakraverty(2009)	19.739	49.348	49.348	79.400	100.17				
Leissa (1973)	19.7392	49.3480	49.3480	78.9568	98.6960	98.6960	128.3049	128.3049	167.7833

Table 5. Frequency parameters for a C-C-C-C square plate.

Solution	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
Current	35.8742	71.3933	71.4026	102.8787	131.0094	131.6766
Li et al.(2004, 2009)	35.986	73.398	73.398	108.24	131.59	132.22
Low et al.(1998)	36.02	73.50	73.50	108.45		
Chakraverty (2009)	35.988	73.398	73.398	108.26	131.89	
Leissa (1973)	35.992	73.413	73.413	108.27	131.64	132.24

Table 6. Frequency parameters for a C-S-S-F square plate.

Solution	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
Current	15.9363	29.6641	51.4547	63.3525	68.4213	102.6084
Li (2004, 2009)	16.785	31.14	51.392	64.016	67.549	101.21
Chakraverty (2009)	16.811	31.173	51.454	65.735	67.794	
Leissa (1973)	16.865	31.138	51.631	64.043	67.646	101.21

boundary condition was readily represented by setting $k_1a^3/D = K_1a/D = k_3a^3/D = k_4a^3/D = 10^{10}$ $\Omega = \omega a^2\sqrt{\rho h/D}$;$K_{3,4} = 0$, $k_2a^3/D = 100$ and finally $K_2a/D = 10$.Frequency parameters were presented for simply supported plate (S-S-S-S, Simple- Simple-Simple-Simple) along all edges in Table 4 for term number M = N = 4. This boundary condition was readily represented by setting $k_{1,2,3,4}a^3/D = 10^{10}$, and $K_{1,2,3,4} = 0$. The accuracy and convergence of the current solution were again demonstrated.

Then, consider a C-C-C-C (Clamped-Clamped-Clamped-Clamped) plate clamped along all edges. Clamped boundary conditions were obtained when the (Clamped-Simple-Elastic-Simple) estimated by using M = N = 4 terms were given in Table 3 with references. This stiffness's for the boundary springs become infinitely large as $k_{1,2,3,4}a^3/D = 10^{10}$ and $K_{1,2,3,4} = 10^{10}$. The first six frequency parameters were presented in Table 5 for the C-C-C-C plate assuming M = N = 4.

Finally, consider a square plate clamped along a C-S-S-F (Clamped-Simple-Simple-Free) square plate. This boundary conditions were created by simply setting the stiffness's of the translational and rotational springs $k_1a^3/D = K_1a/D = 10^{10}$, $k_2a^3/D = k_3a^3/D = 10^{10}$, $K_2 = K_3 = K_4 = 0$ and $k_4 = 0$. The six smallest frequency parameters were given in Table 6 with respect to references.

Conclusion

A set of admissible functions developed by Li has been employed for the free vibrations of rectangular plates with general elastic restraints along all the edges. This proposed study has actually developed a general technique for deriving a complete set of Li's functions. Polynomial-Fourier series solution approach for Levy type plate proposed by Li was adopted for full elastic restrained plates. The plate displacement was expressed as a Fourier sine series plus an auxiliary polynomial. Since each of the series expansions should be truncated to a finite number of terms as M = N = 4 in actual numerical calculations. As a result, several numerical examples were presented to demonstrate the accuracy and reliability of the proposed solution method. It is believed that the proposed method was universally applicable to different boundary conditions, including all the classical cases.

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